Research Article

The Philosophy of Circular Statistical Thinking in Human Cognition, Temporal Constructs & Anthropological Studies

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Abstract: Circular statistical thinking addresses data that exhibit periodicity or directionality, such as angles, times, or compass bearings, and finds broad applications across the social and natural sciences, especially relevant for interdisciplinary fields exploring the cyclical nature of cultural rituals, seasonal festivals, migratory patterns, and human cognition. This research aims to bridge the gap between circular statistical methods and philosophical-anthropological inquiries. The objectives are: (1) to examine philosophical underpinnings of cyclicity in human thought, (2) to apply circular statistical frameworks in anthropological analyses of recurring cultural and social behaviors, and (3) to illustrate how periodicity shapes human cognition and cultural organization. Employing an interdisciplinary methodology, we integrate philosophical reasoning with statistical modeling tailored for circular data. We carry out simulation based case studies and theoretical demonstrations to showcase how circular statistics (e.g., von Mises distribution, phase synchronization) can elucidate periodic behaviors in cultural contexts. Our findings demonstrate that circular statistical thinking offers robust quantitative tools for analyzing cyclical human activities, from seasonal and ritual practices to social synchronization. By highlighting mean directions, dispersion, and synchronization metrics, we reveal how periodic structures inform social cohesion and collective identities. This approach contributes new perspectives on the interplay between statistical modeling, human cognition, and cultural evolution, extending the applicability of circular statistics to broader inquiries into human nature and culture.

Keywords: circular cognition; temporal recurrence; anthropological temporality; circular statistics

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1. Introduction

Circular statistical thinking is a branch of statistics focused on data that is directional or periodic in nature, such as angles, times of day, or compass bearings. Unlike traditional linear data, circular data requires specialized methods that account for its inherent periodicity. Studying such data, often analyzed using circular statistical tools, has significant implications for understanding patterns that repeat over time or directions that play a crucial role in natural and cultural systems.

In philosophical and anthropological contexts, circularity is deeply embedded in human thought and behavior. Many cultural practices, social rituals, and natural phenomena follow cyclic patterns. For example, seasonal festivals, daily routines, and migratory behaviors are all characterized by their periodic nature. Circular statistics provides a mathematical framework to model and understand these patterns, offering a unique perspective on how humans perceive and respond to cyclic events.

This paper aims to extend the understanding of circular statistical thinking into the realms of philosophy and anthropology, highlighting its potential to offer novel insights into human cognition, culture, and behavior. We explore how circular statistical methods can be applied to understand the dynamics of ritualistic practices, cultural cycles, and the cognitive representation of time and direction.

Statistics develops methods to evaluate hypotheses based on empirical data, with its philosophy addressing foundational issues, the interpretation of probability, and the justification of inference methods, thus influencing scientific methodology and the epistemic status of theories (Romeijn, 2014). Lindley (2000) presents statistics as the study of uncertainty, emphasizing that statistical inference relies solely on probability, highlighting the importance of constructing probability models, and connecting inference to decision analysis in science and law. Philosophy covers general principles beyond specific sciences, while statistics focuses on reliable data gathering and inference methods; this chapter explores diverse topics across fields, including causal inference, model selection, foundational questions, statistical paradoxes, and four main statistical paradigms: classical, Bayesian, likelihood-based, and Akaikean statistics (Bandyopadhyay & Forster, 2011).

Suppes (2007) examines the role of statistical concepts in scientific experiments and theories, focusing on uncertainty in astronomy, the application of probability in radioactive decay, and the use of Chi-square tests in biology and social sciences for hypothesis evaluation. Gelman and Shalizi (2013) argue that the success of Bayesian statistics aligns more with hypothetical-deductivism than pure inductivism, emphasizing the importance of model checking and revision beyond Bayesian confirmation theory for both philosophical clarity and practical statistical application.

Starmans (2018) reflects on the fragile yet essential role of truth in human civilization, spanning religion, science, and politics, acknowledging its historical significance in epistemology while addressing modern threats like relativism, disinformation, and ideological distortion. Good (1988) reviews key intersections between statistics and philosophy of science, covering topics like probability types, evidence corroboration, p-values, Bayes/non-Bayes approaches, explicativity, induction, and probabilistic causality. Shweder, Casagrande, Fiske, Greenstone, Heelas, Laboratory of Comparative Human Cognition, and Lancy (1977) offer a cognitive-processing view of magical thinking, proposing it as a universal tendency where adults substitute resemblance for correlation, leading to intuitive but mistaken inductive reasoning in beliefs about relationships between objects and events.

Veissière, Constant, Ramstead, Friston, and Kirmayer (2019) integrate the theories of cultural acquisition using the variational (free- energy) approach, proposing that humans learn culture by inferring others' expectations ("thinking through other minds"), with implications for cognition, enculturation, adaptation, and debates in cognitive science. The anthropology of Christianity has lagged due to anthropological biases toward cultural continuity, which conflict with Christianity's emphasis on radical change, challenging anthropologists to reconsider their models of time, belief, and transformation (Robbins, 2007).

Comparative cognition research using animal models like birds and great apes reveals cognitive traits across species and enhances our understanding of uniquely human spatial, temporal, and numerical cognition (Haun et al., 2010). Kriegeskorte, Lindquist, Nichols, Poldrack, and Vul (2010) outline the ongoing debates on 'circular' or 'nonindependent' analysis in brain imaging, clarifying key points of consensus and disagreement through expert responses to collaboratively formulated questions, with agreed-upon answers summarized in a consensus box.

The economic crisis in Russia and shifting migration policies are reshaping anthropological views on migration, emphasizing Central Asian migrants' circular mobility, deportability, and nostalgia amidst economic volatility and recurrent shifts in post-Soviet migration patterns (Abashin, 2019). Gell (2021) explores the phenomenon of time across cultural, psychological, anthropological, and phenomenological perspectives, analyzing theories from notable thinkers and concluding with a model of social/cognitive time. Challenging the universal Space-Time Mapping Hypothesis, this study of the Amondawa language reveals that spatial terms are not used to express time, suggesting that time concepts are culturally mediated rather than universally structured (Sinha et al., 2011).

Bronowski and Long argue that Zuckerman's anthropological findings diverged from classical comparisons because he analyzed measurements separately instead of using the discriminant function, which more conclusively distinguished between human and chimpanzee dental samples (Yates & Healy, 1951). Kowalski (1972) criticizes the rising use of multivariate methods in anthropometric research, noting limitations in effectively conveying data insights and calling for further theoretical and practical advancements.

Schumm, Crawford, Barkey, Bush, and Bosch (2021) demonstrate how basic statistical methods can systematically address open questions in historical and cultural analyses, using examples from Native American conflicts, New Testament textual variations, and gospel narratives. Cremers and Klugkist (2018) provide a tutorial on analyzing circular data in cognitive psychology and social sciences, covering data inspection, model fitting, estimation, and hypothesis testing with *R*. Anthropologists are increasingly analyzing the cognitive structures underlying social behaviors, using symbolic notation and logical-mathematical models to reveal the abstract principles organizing sociocultural systems, with a focus on areas like kinship, values, and folk science (Wallace, 1962).The novelty of this research lies in its interdisciplinary approach, combining circular statistical thinking with philosophical and anthropological insights. The objective is to establish a deeper connection between the mathematical properties of circular data and human cultural and cognitive processes. Specifically, this research seeks to:

1. Explore the philosophical implications of circular statistical thinking as a model for understanding cyclical human behaviors and cultural practices.
2. Demonstrate the relevance of circular statistical principles in anthropological studies, particularly in understanding cultural rituals, seasonal activities, and social behaviors.
3. Provide a mathematical and philosophical framework for modeling recurring belief systems, periodic decision-making, and cultural adaptation using circular statistical inference.

This approach bridges the gap between statistical methodology and philosophical inquiry, offering a novel perspective on the role of periodicity in shaping human understanding and culture

2. Materials and Methods

This section presents the methodological framework for our study, outlining the research design, data simulation techniques, and statistical methods used to explore circular statistical thinking in human cognition, temporal constructs, and anthropological studies. Given the interdisciplinary nature of this work, we integrate mathematical formalisms with conceptual insights to make the methodology accessible to both statistical and philosophical audiences.

2.1. Research Design: A Statistical-Philosophical Integration

Our research adopts an interdisciplinary approach that bridges circular statistics with philosophical and anthropological discourse. The foundation of this study lies in the observation that many human cognitive and cultural phenomena exhibit cyclicality—be it the perception of time, seasonal rituals, or periodic decision-making. Circular statistics provides a natural framework to analyze such cyclic behaviors mathematically, allowing us to formalize the recurrence and synchronization of cultural and cognitive events.

The research is structured as follows:

1. Conceptual exploration that inherently follow circular structures, such as the cyclic nature of festivals, habitual decision-making, and social synchronization.
2. Mathematical representation is mapped onto circularstatistical frameworks, primarily using the von Mises distribution, phase analysis, and synchronization metrics.
3. Simulation and analysis simulate data that mimics real-world cyclic phenomena and analyze them using circular statistical tools to reveal underlying patterns in periodic human behavior.
4. Interpretation and theoretical synthesis are interpreted in the context of philosophy and anthropology, examining how statistical results correspond to theoretical constructs such as the perception of time, ritualistic continuity, and cultural adaptation.

By combining rigorous statistical methods with philosophical inquiry, we aim to demonstrate the relevance of circular statistics beyond its traditional applications in physical sciences and into the broader domain of human thought and culture.

2.2. Mathematical Foundations of Circular Statistics

Circular data differ fundamentally from linear data because they exhibit periodicity, meaning that the values wrap around a fixed interval (typically ($[0,2\pi)$. This has profound implications for analyzing human behavior, as many cognitive and cultural processes unfold in cycles rather than along an unbounded linear continuum.

The mean direction $\underline{\theta}$ of a set of circular observations $\{\theta_1, \theta_2, \ldots, \theta_n\}$ is a critical measure in understanding the central tendency of cyclic phenomena.

2.2.1. Von Mises Distribution: Modeling Cyclic Phenomena

One of the fundamental models for circular data is the von Mises distribution, given by:

$$vM(\theta; \mu, \kappa) = \frac{e^{\kappa\cos(\theta-\mu)}}{2\pi I_0(\kappa)} \tag{1}$$

where:

- μ represents the mean direction, analogous to the central tendency of cyclic phenomena.

- $\varkappa$ is the concentration parameter, indicating how tightly observations cluster around μ. A high $\varkappa$ suggests strong periodicity (e.g., a fixed religious festival date), while a low $\varkappa$ suggests flexibility (e.g., a shifting cultural practice).
- $I_0(\varkappa)$ is the modified Bessel function of the first kind of order zero, ensuring the distribution normalizes properly.

The von Mises distribution captures the probabilistic nature of cyclic human behaviors. For instance, the recurrence of social gatherings at particular times of the year can be modeled using $\varkappa$, providing a measure of adherence to tradition. If a festival has been celebrated around the same time each year for centuries, a high $\varkappa$ value would reflect this regularity. Conversely, societies undergoing cultural shifts may show a broader spread in their ritual timing, corresponding to a lower $\varkappa$.

2.2.2. Mean Direction and Cultural Consistency

The mean direction of circular data is given by:

$$\bar{\theta} = atan2\left(\sum_{i=1}^{n} sin\,(\theta_i)\ ,\sum_{i=1}^{n} cos\,(\theta_i)\right) \tag{2}$$

Philosophically, the mean direction represents a community's collective inclination toward certain periodic behaviors. In anthropological studies, it serves as a statistical analog to the most common time of occurrence for a social or cognitive cycle. If we consider human sleep-wake rhythms, for example, the mean direction of sleep onset across a population could indicate the average habitual bedtime, with variations reflecting differing lifestyles and environmental influences.

2.2.3. Circular Variance and the Fluidity of Rituals

The spread of circular data is captured by the circular variance:

$$V_c = 1 - R,$$

$$R = \frac{1}{n}\sqrt{\left(\sum_{i=1}^{n} cos\,(\theta_i)\right)^2 + \left(\sum_{i=1}^{n} sin\,(\theta_i)\right)^2} \tag{3}$$

A small circular variance implies that events are tightly clustered around the mean direction, indicating strong periodic adherence (e.g., a community that consistently begins agricultural planting on the same date every year). A high variance, on the other hand, suggests variability in the recurrence of events, potentially revealing cultural flexibility or external influences that disrupt periodic traditions.

2.2.4. Phase Analysis and Temporal Synchronization

One of the most insightful aspects of circular statistics in human studies is phase analysis, which measures the temporal alignment between different cyclic processes. The relative phase difference between two recurring events θ*i* and θ*j* is given by :

$$\Delta\phi_{ij} = \theta_j - \theta_i \ mod\ 2\pi \tag{4}$$

This metric allows us to examine whether different cultural or cognitive cycles align or diverge over time. If two communities historically celebrated a solstice festival on the same day but have diverged due to social or political shifts, their phase difference would increase. In cognition, phase differences in neural oscillations could relate to synchronization in collective decision-making or group coordination.

2.2.5. Synchronization and Social Cohesion

In human societies, synchronized behaviors—such as communal chanting, synchronized rituals, or shared work schedules – enhance social cohesion. The degree of synchronization across a population can be captured using the order parameter:

$$R(t) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i(t)}\right| \tag{5}$$

where *N* represents the number of individuals or groups under consideration. A high *R(t)* suggests collective alignment in periodic behaviors, while a low *R(t)* indicates greater dispersion and potential fragmentation in social synchronization. This measure has direct applications in understanding how rituals strengthen social bonds. In religious practices, synchronized movements (e.g., mass prayer timings) may reflect high *R(t)*, reinforcing communal identity. Conversely, declining *R(t)* could indicate weakening traditions, as seen in modern societies where individuals increasingly follow personalized rather than collective schedules.

2.3. Simulation and Data Analysis

To illustrate these concepts, we conduct a simulation study where we generate multiple datasets representing different cyclic human behaviors:

1. *Fixed cultural rituals:* Simulated data where $\varkappa$ is high, reflecting tightly clustered events around a specific date.

2. *Flexible social practices:* Data with moderate $\varkappa$, allowing some temporal variation in recurring behaviors.

3. *Diverging cognitive patterns:* Two groups with different mean directions μ_1, μ_2 and increasing phase difference, simulating cultural divergence.

4. *Social synchronization over time:* Tracking *R(t)* as individuals adopt or abandon synchronized cultural practices.

Analyzing these datasets, we demonstrate how circular statistics quantifies the evolution of periodic human behaviors and provides insights into cultural stability, cognitive rhythm regulation, and the synchronization of collective rituals.

2.4. Philosophical and Anthropological Implications

The application of circular statistics in human cognition and cultural studies extends beyond mathematical convenience. It provides a structured way to measure and interpret fundamental questions about temporal perception, social cohesion, and the persistence or transformation of traditions. By quantifying periodicity, we bridge the gap between statistical analysis and philosophical reflections on the cyclic nature of human existence.

3. Circular Statistical Thinking in Philosophical Studies

Circular statistical thinking provides a mathematical lens through which we can analyze the philosophical implications of cyclicality in human experience. In philosophical discourse, the notion of cyclicity is omnipresent, particularly in the study of time, consciousness, and the recurrence of events. Circular statistics, with its inherent focus on periodicity, offers a framework to model these recurring phenomena, allowing us to formalize concepts such as the eternal return, the perception of time, and the interconnectedness of events in a continuous cycle.

The fundamental element of circular statistical thinking is the unit circle, where angular measurements provide a natural representation of periodic data. Let us denote a circular variable $\theta \in [0, 2\pi)$, representing a point on the unit circle $S^{\mathbb{1}}$. Philosophically, θ can be interpreted as the state of a recurring phenomenon such as a season, a ritual, or a mental state that repeats in a consistent cycle. The mathematical treatment of such a variable involves defining the mean direction and assessing the dispersion of the data points around the circle.

3.1. Mean Direction and the Concept of Recurrence

The mean direction $\bar{\theta}$ of a set of circular observations $\{\theta_1, \theta_2, \ldots, \theta_n\}$ is a critical measure in understanding the central tendency of cyclic phenomena. According to Jammalamadaka and SenGupta (2001), it is computed as:

$$\bar{\theta} = atan2\left(\sum_{i=1}^{n} sin\,(\theta_i)\ , \sum_{i=1}^{n} cos\,(\theta_i)\right) \qquad (6)$$

where *atan2* is the two-argument arctangent function that ensures the correct quadrant for the resultant angle. Philosophically, $\bar{\theta}$ represents the "center" of a recurring phenomenon analogous to the average occurrence of a cyclic event, such as the peak of a cultural festival or the habitual return of a particular mental state. This notion of mean direction resonates with the philosophical concept of the "eternal return" – the idea that all events are destined to repeat indefinitely. In this context, the meandirection quantitatively represents the central aspect of recurring events. If we consider human experience as inherently cyclical, $\bar{\theta}$ indicates the equilibrium point around which experiences oscillate, thus providing insight into the stability or regularity of our cyclical nature.

3.2. Circular Variance and Dispersion of Beliefs

The circular variance Vc measures the dispersion of circular data, providing insight into the concentration of the observations around the mean direction. According to Jammalamadaka and SenGupta (2001), it is given by:

$$V_c = 1 - R \tag{7}$$

where R is the mean resultant length defined as:

$$R = \frac{1}{n}\sqrt{\left(\sum_{i=1}^{n} cos\ (\theta_i)\right)^2 + \left(\sum_{i=1}^{n} sin\ (\theta_i)\right)^2} \tag{8}$$

A low circular variance implies that the observations are tightly clustered around the mean direction, which suggests a high level of consistency in the recurrence of the phenomenon being studied. In philosophical terms, a low Vc may indicate the deterministic nature of a cycle – where experiences or events recur with high regularity, reflecting a sense of inevitability.

Conversely, a high circular variance implies greater unpredictability and a more scattered distribution of events around the cycle. This aligns with the philosophical notion of free will or randomness within the context of cyclicality, where there is greater variability in the recurrence of events, thus allowing for divergence from a predetermined path.

Figure 1 shows the circular mean direction and variance for the generated samples. The red line represents the mean direction and captures the cyclic events' central tendency. The circular variance measures the dispersion around the mean, providing insights into the consistency of periodic cultural practices or rituals.

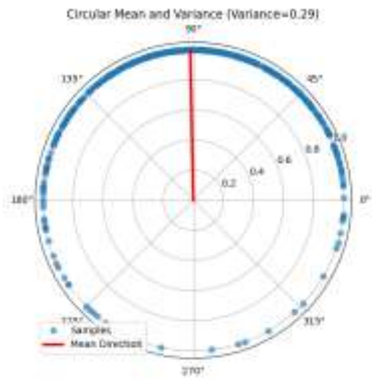

Figure 1. Circular mean and variance of generated samples.

Note: The red line represents the mean direction, indicating the central tendency of circular data.

3.3. Von Mises Distribution and the Probabilistic Nature of Recurrence

The von Mises distribution, often called the "circular normal distribution," plays a key role in modeling the probabilistic nature of cyclic phenomena. According to Jammalamadaka and SenGupta (2001), the probability density function of the von Mises distribution is given below:

$$vM(\theta; \mu, \kappa) = \frac{e^{\kappa coscos\ (\theta - \mu)}}{2\pi I_0(\kappa)} \tag{9}$$

where μ is the mean direction, ϰ is the concentration parameter, and I_0 (ϰ) is the modified

Bessel function of the first kind of order zero. Philosophically, μ represents the most probable direction of recurrence, while $\varkappa$ quantifies the strength of this recurrence

The von Mises distribution allows us to model the uncertainty inherent in the recurrence of events. In human experience, while certain events may recur (e.g., the changing of seasons or habitual behaviors), there is always an element of unpredictability. The parameter $\varkappa$ captures this uncertainty: a high $\varkappa$ indicates that events recur with high certainty and regularity, while a low $\varkappa$ suggests greater randomness in their occurrence.

This probabilistic interpretation of recurrence aligns with the philosophical debate between determinism and indeterminism. Circular statistical thinking, through the von Mises distribution, offers a framework to understand the balance between the deterministic recurrence of events and the inherent uncertainty that characterizes human experience.

We illustrate the behavior of the von Mises distribution with different concentration parameters ($\varkappa$). The concentration parameter $\varkappa$ controls how tightly the data is clustered around the mean direction μ. Higher values of $\varkappa$ indicate stronger concentration, akin to lower variance in linear data.

Figure 2 shows the von Mises PDF plotted for three different concentration parameters ($\varkappa$ = 0.5, 2, 10). The plot also overlays scatter plots of simulated datasets drawn from them corresponding von Mises distributions. The samples are represented at different radii to avoid overlap, making comparing the sample distributions across different values of $\varkappa$ easier. The radii are set to 0.6, 0.8, and 1.0 for $\varkappa$ = 0.5, 2, and 10, respectively. Different markers and colors represent the scatter points for a clear distinction.

The overlay of the von Mises PDF with the simulated data provides an insightful visualization of how the concentration parameter affects the distribution. For $\varkappa$ = 0.5, the distribution is quite spread out, indicating low concentration, whereas for $\varkappa$ = 10, the distribution is highly concentrated around the mean direction. The simulated data points align well with the corresponding PDFs, demonstrating the relationship between the theoretical distribution and the observed samples.

This visualization helps understand the philosophical context of circular data, where the concentration parameter $\varkappa$ can be interpreted as the degree of conformity or adherence to a central tendency. Higher $\varkappa$ values may represent stronger cultural adherence to certain rituals or behaviors, whereas lower $\varkappa$ values could indicate more variability or openness to diverse behaviors.

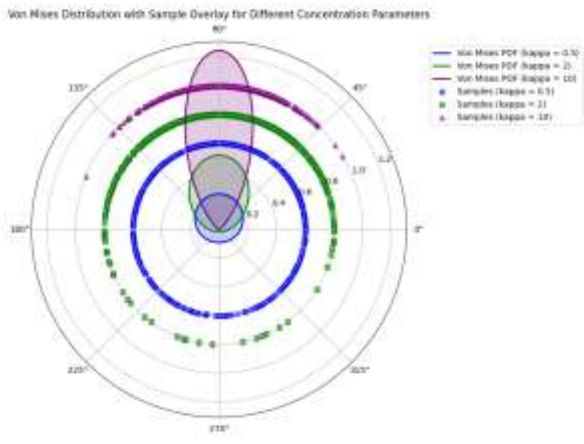

Figure 2. Von Mises distributions for different concentration parameters ($\varkappa$ = 0.5, 2, 10) with overlay of simulated datasets.

Note: The scatter points are positioned at different radii to visualize the sample distributions. The figure demonstrates how the concentration parameter affects the clustering of data around the mean direction.

3.4. Phase Analysis and the Interconnectedness of Events

Another important concept in circular statistics is phase analysis, which involves examining the relative positioning of different events on the unit circle. Let $\theta_1, \theta_2, \ldots, \theta_n$ represent different cyclic phenomena, such as seasonal rituals, cultural practices, or recurring thoughts. The relative phase difference between two events θ_i and θ_j can be represented as:

$$\Delta\phi_{ij} = \theta_j - \theta_i \bmod 2\pi \qquad (10)$$

Phase analysis allows us to explore the interconnectedness of different cyclic events, providing insights into how one event may influence or align with another. In philosophical terms, this interconnectedness reflects the holistic nature of human experience, where different aspects of life are interdependent and influence each other in a continuous cycle. The concept of phase synchronization, where multiple cyclic events become aligned, can be seen as a metaphor for harmony and coherence in human existence.

4. Circular Statistical Thinking in Anthropological Studies

Circular statistical thinking offers a valuable framework for examining anthropological phenomena, particularly in understanding the cyclical nature of cultural practices, social behaviors, and human-environment interactions. The anthropological domain is rich with examples of periodicity – ranging from seasonal festivals and traditional rituals to migratory patterns and social behaviors. The application of circular statistical methods provides a quantitative foundation to analyze and interpret these phenomena, which are often shaped by environmental, social, and cultural factors.

4.1. Seasonal and Ritualistic Practices

One of the fundamental aspects of anthropological research is the study of rituals and cultural practices, many of which exhibit clear cyclical patterns. Consider an annual festival celebrated by a community. Let θ_i denote the timing of the festival in year i, represented as an angle on the unit circle, where $\theta_i \in [0, 2\pi)$. By treating these occurrences as circular data points, we can employ statistical measures such as the mean direction μ and the concentration parameter $\varkappa$ to understand the regularity and social significance of the ritual

The timing of these events can be modeled using the von Mises distribution, which provides a way to quantify the concentration of the festival occurrences around the mean direction. Here μ represents the average timing of the event, and $\varkappa$ indicates the level of adherence to tradition. A high value of $\varkappa$ suggests that the community celebrates the festival with high regularity, reflecting a deep cultural significance and the importance of adhering to traditional practices. Conversely, a lower $\varkappa$ may indicate flexibility in the timing of the ritual, possibly due to social or environmental factors that necessitate adaptation.

The use of circular variance Vc further provides insights into the variability of the festival's occurrence:$Vc = 1 - R$, where R is the mean resultant length that measures the clustering of the observations, low circular variance suggests a strong cultural commitment to the ritual. In contrast, a high variance may imply a changing cultural landscape where traditional practices are being modified or losing significance.

4.2. Migration Patterns and Human Mobility

Migration is another important anthropological phenomenon that can be effectively analyzed using circular statistical methods. Human migratory patterns often exhibit seasonality, driven by environmental cues such as temperature, resource availability, or climatic conditions. Let θi represent the direction of migration for a population in year i, where θ_i is measured in degrees relative to a reference point (e.g., north). The concentration of migration directions can be analyzed using circular mean and variance to determine the stability and directionality of migratory routes.

The von Mises distribution can also be employed to model the probabilistic nature of migration directions. By estimating the mean migration direction μ and the concentration parameter $\varkappa$, anthropologists can gain insights into the environmental factors influencing migration and the degree of cohesion within migrating groups. For instance, a high $\varkappa$ value might suggest strong environmental pressures that drive a consistent migratory direction. In contrast, a lower $\varkappa$ value could indicate variability in migration routes due to changing environmental or social conditions. Moreover, phase analysis can be applied to study the synchronization of migration across different communities or species. The relative phase difference $\Delta\phi_{ij}$ between the migration of two populations i and j can be represented as $\Delta\phi_{ij} = \theta_j - \theta_i \mod 2\pi$, which provides a measure of the temporal alignment between migratory behaviors. Such phase relationships can reveal the extent to which different communities are influenced by shared environmental cues or social interactions, thereby offering a deeper understanding of the interconnectedness of migratory behaviors.

4.3. Cultural Synchronization and Social Cohesion

Circular statistical thinking is also crucial in understanding social cohesion and cultural synchronization within human societies. Social practices, such as communal dances, religious observances, and collective decision-making, often exhibit cyclical patterns that can be analyzed using circular statistics. The concept of phase synchronization, commonly used in the study of coupled oscillatory systems, can be extended to anthropological studies to explore how different social groups achieve coherence in their collective behaviors.

Let $\theta i(t)$ represent the phase of social practice for group i at time t, where $\theta i(t)$ evolves on the unit circle. Phase synchronization occurs when the phase difference between two groups remains approximately constant over time, which can be expressed as:

$$\Delta\phi ij(t) = \theta j(t) - \theta i(t) \approx \text{const}, \tag{11}$$

for all t within a given period. This condition implies that the social practices of groups i and j are synchronized, reflecting a high degree of social cohesion and cultural integration. Such synchronization can indicate strong cultural ties, effective communication, and shared values within the community.

The strength of synchronization can be quantified using the order parameter R(t), defined as:

$$R(t) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i(t)}\right| \tag{12}$$

where N is the number of groups under consideration. The order parameter ranges from 0 to 1, where $R(t) = 1$ indicates perfect synchronization (i.e., all phases are identical), and $R(t) = 0$ represents complete desynchronization. High values of $R(t)$ suggest that the community exhibits a high degree of coordination in its cultural practices, whereas low values indicate diversity and potential fragmentation within social behaviors.

5. Implications for Human Cognition and Cultural Understanding

Circular statistical thinking offers a unique framework for understanding human cognition and cultural evolution, particularly in how individuals and societies perceive, process, and react to cyclic phenomena. Cyclicity, as embodied by circular data, is inherent in the daily lives of individuals – from biological rhythms such as sleep-wake cycles to cultural phenomena such as annual rituals. In this section, we explore how circular statistical methods provide insight into the cognitive processes of perceiving time, the cultural construction of temporal cycles, and the collective behaviors that shape human societies.

5.1. Cognitive Representation of Circular Concepts

The human mind is adept at perceiving and processing periodic phenomena, which is critical for survival and adaptation. Many cognitive processes rely on recognizing temporal patterns, such as circadian rhythms, which can be represented as circular variables. Consider the sleep-wake cycle, where the time of sleep onset θ_i can be modeled as a circular variable on the interval $[0, 2\pi)$. The circular mean θ of the sleep onset times for an individual over multiple days can be computed by the circular statistical formulae:

$$\bar{\theta} = atan2(\textstyle\sum_{i=1}^{n} sin\,(\theta_i)\ ,\sum_{i=1}^{n} cos\,(\theta_i)\,) \tag{13}$$

where n is the number of observations. The consistency of sleep onset times, which reflects the regularity of an individual's circadian rhythm, can be quantified using the circular variance: $Vc = 1 - R$, where R is the mean resultant length. A low circular variance indicates a stable and well-regulated sleep-wake cycle essential for optimal cognitive functioning. Conversely, a high circular variance may indicate disruptions in the circadian rhythm, which can negatively impact cognition and overall well-being.

From a philosophical perspective, the perception of time itself can be viewed through the lens of circularity. As perceived by individuals, the cyclical nature of time is inherently linked to natural periodicities, such as the rotation of the Earth. Circular statistical thinking provides a quantitative framework to model this perception, allowing us to understand how temporal cycles influence cognitive processes, such as memory formation, decision-making, and the anticipation of future events.

5.2. Temporal Constructs and Cultural Evolution

In anthropological studies, cultural constructs of time are deeply embedded in the cyclic nature of human existence. Calendars, festivals, and agricultural cycles are all manifestations of the human need to organize and interpret temporal experiences. The construction of these temporal cycles can be analyzed using circular statistical tools to understand their evolution and cultural significance.

Let θ_i represent the timing of a cultural festival within a given year, measured as an angle on the unit circle. By employing the von Mises distribution, we can model the probability density of the festival's occurrence: vM(θ; μ, ϰ), where μ represents the average timing of the festival and ϰ reflects the concentration of the timing around μ. A high value of ϰ suggests that the festival is celebrated with high regularity, reflecting its importance within the cultural framework. The periodic nature of these cultural events reinforces a collective understanding of time and fosters social cohesion by synchronizing the behaviors of individuals within the community.

The role of circular statistical thinking extends beyond merely representing cultural practices; it provides a means to understand cultural adaptation and resilience. For example, the variability in the timing of cultural events, as captured by circular variance, can offer insights into how communities adapt to environmental changes or social disruptions. High variance in the timing of cultural events may indicate flexibility and adaptability, allowing cultures to respond dynamically to changing conditions, whereas low variance suggests a rigid adherence to tradition.

5.3. Collective Behaviors and Phase Synchronization

Collective human behaviors, such as communal dances, rituals, and coordinated actions, often exhibit cyclicity that can be studied using the concept of phase synchronization. Phase synchronization refers to the alignment of cyclic activities among individuals or groups, which can be quantified using circular statistical measures. Let *θi(t)* represent the phase of an individual's participation in a collective behavior at time t, where *θi(t)* is defined on the unit circle.

The phase difference can express the synchronization between individuals *i* and *j*:

$$\Delta\phi ij(t) = \theta j(t) - \theta i(t) \bmod 2\pi \tag{14}$$

Phase synchronization occurs when *Δϕij(t)* remains approximately constant over time, indicating that individuals are in sync with each other. The overall level of synchronization within a group can be quantified using the order parameter *R(t)*, defined as:

$$R(t) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i(t)}\right| \tag{15}$$

where *N* is the number of individuals in the group. The value of *R(t)* ranges from 0 to 1, where *R(t)* = 1 indicates perfect synchronization, and *R(t)* = 0 represents complete desynchronization. High values of *R(t)* suggests a strong alignment of collective behaviors, which can foster social cohesion and reinforce cultural identity.

The concept of phase synchronization has significant implications for understanding social dynamics and cultural integration. In anthropological contexts, synchronized behaviors, such as group rituals and coordinated dances, reinforce social bonds and establish a shared cultural identity. By modeling these behaviors using circular statistical methods, we can gain insights into the mechanisms underlying social cohesion, cultural resilience, and the emergence of collective identities.

6. Simulation Study

This section presents a detailed simulation study to illustrate the above concepts. The simulation was conducted using the von Mises distribution to model circular data, demonstrating key aspects such as the concentration of angular data, phase analysis between different datasets, and synchronization using order parameters. The plots generated visually represent these concepts and explain their philosophical and anthropological significance.

6.1. Phase Analysis Between Circular Datasets

Phase analysis is a powerful tool for examining the relationship between different cyclic phenomena. We generated two datasets with different mean directions and analyzed their phase differences. The relative phase difference between two events θ_i and θ_j is calculated as:

$$\Delta\phi ij = (\theta j - \theta i) \bmod 2\pi \tag{16}$$

Figure 3 shows the distribution of phase differences between the two datasets. The histogram illustrates how closely aligned or misaligned the two cyclic events are, which can have implications for understanding synchronization or divergence in cultural practices. Also, see figure 4. The visual representation highlights the differences and potential divergence in cyclic cultural phenomena.

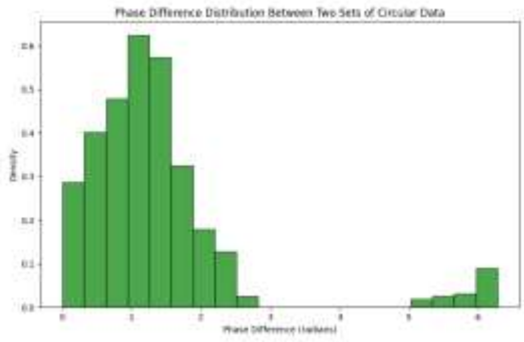

Figure 3. Histogram of phase differences between two sets of circular data.
Note: This provides insights into the alignment and divergence of cyclic cultural practices.

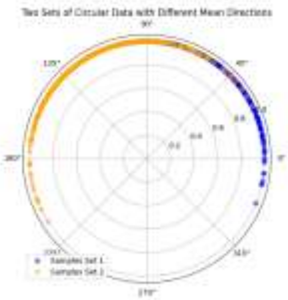

Figure 4. Polar plot of two sets of circular data with different mean directions.

The alignment of the two datasets is further visualized in Figure 4, where both datasets are plotted on a polar plot. The differences in mean direction reflect potential diversity in cultural practices, which may vary depending on environmental or social influences.

6.2. Illustration of the Concept of Cultural Synchronization and Social Cohesion within Human Societies

To illustrate the concept of cultural synchronization and social cohesion within human societies, we conducted a simulation study using circular statistical techniques. In this simulation, we modeled the evolution of phases representing social practices for multiple groups over time. Let $\theta i(t)$ denote the phase of social practice for group i at time t, where $\theta i(t)$ evolves on the unit circle.

This simulation aims to understand how different social groups achieve coherence in their collective behaviors, which can be analyzed by examining the phase synchronization between groups. Phase synchronization is said to occur when the phase difference between groups remains approximately constant over time:

$$\Delta\phi ij(t) = \theta j(t) - \theta i(t) \approx \text{const} \quad (17)$$

for all t within a given period. This indicates a high level of cultural integration and social cohesion.

The strength of synchronization can be quantified using the order parameter R(t):

$$R(t) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i(t)}\right| \quad (18)$$

where N is the number of groups. The order parameter $R(t)$ ranges from 0 to 1, where $R(t) = 1$ indicates perfect synchronization, and $R(t) = 0$ represents complete desynchronization.

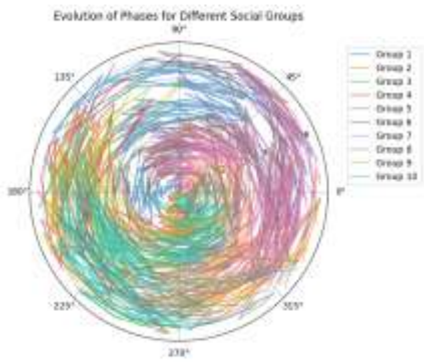

Figure 5. Evolution of phases for different social groups over time.

Note: The plot illustrates the dynamic changes in the phases of social practices, highlighting moments of alignment and divergence between groups. The radial coordinate represents time.

Figure 5 shows the evolution of phases for each social group over time. Each group starts with a random initial phase, and the evolution of these phases demonstrates the cyclic nature of social practices, such as communal dances or religious rituals. The figure highlights how the phases evolve on the unit circle, showing moments of alignment and divergence.

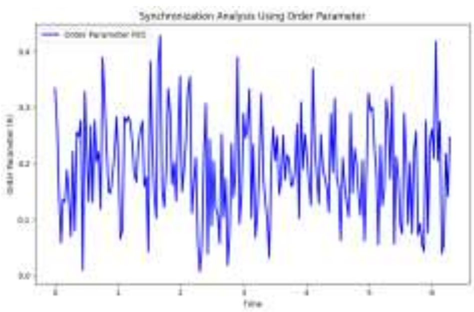

Figure 6. Order parameter *R(t)* over time, illustrating the synchronization of phases across different social groups.

Note: High values of *R(t)* indicate strong cultural cohesion, whereas low values suggest divergence in social practices

Figure 6 shows the order parameter *R(t)* over time, representing the degree of synchronization among the groups. High *R(t)* values indicate strong cultural synchronization, suggesting that the social groups are in harmony and exhibit coordinated behavior. Low values of *R(t)* indicate potential fragmentation, with the groups showing variability in their cultural practices.

The simulation visually represents how cultural synchronization can emerge within communities. Social cohesion can be observed when social practices exhibit cyclical patterns and phases align. This reflects the shared values, effective communication, and collective identity contributing to a synchronized society. Conversely, moments of low synchronization may represent diversity or differences in cultural expressions, which could either enrich the community or indicate social fragmentation.

6.3. Synchronization Analysis Using Order Parameters

The concept of synchronization is fundamental in understanding collective cultural behavior. The order parameter *R(t)* quantitatively measures the synchronization level among individuals or groups. The order parameter is defined as:

$$R(t) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i(t)}\right| \qquad (19)$$

where N is the number of individuals or groups, and $\theta i(t)$ represents the phase of each individual at time t. A high value of $R(t)$ indicates strong synchronization, whereas a low value suggests desynchronization.

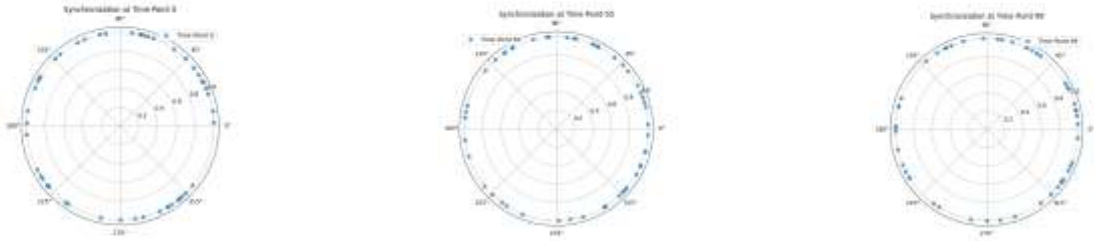

Figure 7. Polar representation of synchronization at different time points.
Note: These plots provide insights into the evolution of synchronization among groups over time

The plot of the order parameter over time, illustrating the level of synchronization among groups, is similar to Figure 6, demonstrating how synchronization evolves and can reflect changes in collective cultural practices over time.

To further understand synchronization, we visualized the phase distribution of individuals at specific time points (Figure 7). These polar plots illustrate the extent to which phases are aligned or dispersed, providing insights into the dynamics of social cohesion and cultural integration.

7. Discussion

Circular statistical thinking has traditionally been applied in biological and physical contexts, such as analyzing animal movement paths or wind directions. A key novelty of the present study lies in extending these statistical tools to philosophical questions about recurrence and anthropological investigations into culture and society. By emphasizing the cyclical nature of social and cognitive phenomena, we bring fresh quantitative insights to disciplines that frequently rely on qualitative assessment.

First, our work highlights the mean direction and circular variance as critical measures to interpret the stability and variability in periodic behaviors. Philosophically, these metrics bridge discussions of determinism (low variance) and spontaneity (high variance) within cyclical recurrences. Anthropologically, they help compare how rigidly or flexibly communities maintain traditions, rituals, or festivals. This dual interpretation underscores the broad relevance of circular statistics: beyond pure data analysis, it becomes a lens through which we can view human choices, freedoms, and constraints in the face of cyclical structures.

Second, applying phase analysis reveals how different cyclical behaviors may align or diverge over time. This tool is relatively underused in anthropology yet offers considerable insight. For instance, understanding how two rituals from different communities align (small phase difference) or systematically diverge (significant phase difference) can inform us about cultural exchange, syncretism, or socio-political factors driving communities apart. Moreover, from a cognitive standpoint, phase alignment might mirror shared mental models or collective memories in a group—indicating deeper synchronization in how individuals perceive time, tradition, and communal identity.

Third, the order parameter concept, borrowed from the study of coupled oscillatory systems, speaks directly to anthropologists studying social cohesion. While anthropological theory has long recognized the importance of ritual synchronization—such as Emile Durkheim's notion of collective effervescence—our quantitative metrics offer a more precise way to capture and compare synchronization across contexts or over historical timelines. High-order parameters suggest a coherent, unified community, whereas a decline could signal fragmentation or the rise of subcultures with distinct cyclical practices. This opens up possibilities for cross-cultural comparisons and generates robust empirical measures that can be tracked longitudinally.

Despite these new insights, certain outcomes have not been fully achieved in the current study. One limitation is the translation from theoretical models to extensive field data. Real-world applications could involve complexities like irregular sampling, incomplete ritual observations, or conflicting time scales (e.g., lunar vs. solar calendars). Future research should refine sampling strategies and incorporate advanced circular modeling techniques (e.g., Bayesian hierarchical approaches) to handle partial or noisy observations. Also, we have not delved deeply into the intersection of circular statistics with network analysis—an area that could enrich the study of social connectivity and cultural exchange patterns.

Quantitative rigor in understanding periodic cultural and social phenomena is paramount in an era where global interconnectedness is reshaping traditional cycles. Migration patterns, climate change, and the transformation of cultural calendars under globalization require robust analytic methods. Circular statistics provides a systematic approach to track, compare, and predict changes in these cycles, offering anthropologists, sociologists, and policymakers tools to formulate adaptive strategies. Simultaneously, from a philosophical vantage point, quantifying concepts like recurrence or cyclical inevitability encourages deeper reflections on how societies construct the concept of time.

The present study suggests that interdisciplinary research spanning statistics, philosophy, and anthropology can greatly benefit from a circular perspective on time and events. The metrics we have described – mean direction, circular variance, phase analysis, and synchronization – are not merely mathematical formalities; they directly inform theoretical discussions about the collective nature of cultural memory, the fragility of communal rituals under social change, and the interplay between deterministic structures (e.g., seasonal constraints) and human agency (e.g., flexible festival scheduling). More broadly, institutions aiming to preserve cultural heritage might use these metrics to document and compare how festivals or communal celebrations shift over generations, guiding interventions that maintain communal coherence while allowing for cultural adaptation.

8. Conclusions

Circular statistical thinking provides a robust framework for analyzing and interpreting cyclic patterns in human cognition, cultural rituals, and social behaviors. By conceptualizing cultural practices, rituals, and cognitive processes as circular phenomena, this study highlights how statistical reasoning can be extended beyond its traditional applications to offer deeper insights into human experience. The integration of circular statistics with philosophical and anthropological perspectives enables a more comprehensive understanding of the repetitive and periodic nature of cultural and cognitive structures.

One of the key outcomes of this research is the demonstration that circular statistics can effectively model patterns that conventional linear approaches fail to capture. Many human behaviors – such as religious ceremonies, seasonal festivities, linguistic patterns, and decision-making cycles – exhibit periodicity, making them well-suited for analysis using directional data techniques. By applying statistical models tailored for circular data, we establish a methodological foundation for studying these recurring patterns with greater precision and clarity.

Furthermore, this study underscores the philosophical implications of circular statistical thinking. The notion of time, recurrence, and continuity in human culture aligns with fundamental philosophical inquiries into existence and cognition. Through this lens, circular statistics do not merely serve as a mathematical tool but also as a conceptual bridge between statistical reasoning and philosophical thought, enriching our understanding of human perception and behavior.

From an anthropological perspective, the findings contribute to the study of cultural evolution and social organization by offering a structured approach to examining how traditions and collective behaviors are shaped by cyclic forces. By applying circular statistical models, researchers can better analyze how societies maintain, adapt, and transmit cultural knowledge over time.

Overall, this research advances interdisciplinary scholarship by positioning circular statistical thinking as a critical tool for exploring the dynamics of human cognition and culture. The findings encourage further applications of circular statistics in humanities and social sciences, paving the way for innovative methodologies in studying cyclic phenomena in human life. Future research may expand on these insights by incorporating empirical case studies and refining statistical models to enhance their applicability across diverse cultural and cognitive contexts

Code Availability: We kept this paper's source code and supplementary materials in the GitHub repository, including Python Notebooks, with detailed implementations of the methods discussed in this study. The code repository URL: https://github.com/debashisdotchatterjee/Circular-Statistical-Thinking-in-Human-Cognition-Temporal-Constructs-Anthropological-Studies_Part_1

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References

Abashin, S. N. (2019). Returning home and circular mobility: How crises change the anthropological view of migration. *Anthropology & Archeology of Eurasia, 58*(3), 155–168. https://doi.org/10.1080/10611959.2019.1686902

Bandyopadhyay, P. S., & Forster, M. R. (2011). Philosophy of statistics: An introduction. *Philosophy of statistics, 7*, 1–50. DOI: 10.1016/B978-0-444-51862-0.50001-0

Cremers, J., & Klugkist, I. (2018). One direction? a tutorial for circular data analysis using r with examples in cognitive psychology. *Frontiers in psychology, 9*, 2040. https://doi.org/10.3389/fpsyg.2018.02040

Gell, A. (2021). *The anthropology of time: Cultural constructions of temporal maps and images.* Routledge. https://doi.org/10.4324/9781003135180

Gelman, A., & Shalizi, C. R. (2013). Philosophy and the practice of bayesian statistics. *British Journal of Mathematical and Statistical Psychology, 66*(1), 8-38. https://doi.org/10.1111/j.2044-8317.2011.02037.x

Good, I. (1988). The interface between statistics and philosophy of science. *Statistical Science, 3*(4), 386–397. DOI: 10.1214/ss/1177012754

Haun, D. B., Jordan, F. M., Vallortigara, G., & Clayton, N. S. (2010). Origins of spatial, temporal and numerical cognition: Insights from comparative psychology. *Trends in Cognitive Sciences, 14*(12), 552–560. https://doi.org/10.1016/j.tics.2010.09.006

Jammalamadaka, S. R., & SenGupta, A. (2001). *Topics in Circular Statistics.* World Scientific.

Kowalski C. J. (1972). A commentary on the use of multivariate statistical methods in anthropometric research. *American journal of physical anthropology, 36*(1), 119–132. https://doi.org/10.1002/ajpa.1330360114

Kriegeskorte, N., Lindquist, M. A., Nichols, T. E., Poldrack, R. A., & Vul, E. (2010). Everything you never wanted to know about circular analysis, but were afraid to ask. *Journal of cerebral blood flow and metabolism : official journal of the International Society of Cerebral Blood Flow and Metabolism, 30*(9), 1551–1557. https://doi.org/10.1038/jcbfm.2010.86

Lindley, D. V. (2000). The philosophy of statistics. *Journal of the Royal Statistical Society. Series D (The Statistician), 49*(3), 293-337.

Robbins, J. (2007). Continuity thinking and the problem of christian culture: belief, time, and the anthropology of christianity. *Current anthropology, 48*(1), 5-38. https://doi.org/10.1086/508690

Romeijn, J.-W. (2014). Philosophy of statistics. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (pp. 1-86) http://plato.stanford.edu/archives/win2014/entries/statistics/

Schumm, W., Crawford, D., Barkey, P., Bush, D., & Bosch, D. (2021). Using statistics to analyze anthropological/religious issues from the distant past. *Insights of Anthropology, 5*(1), 337–346. DOI: 10.36959/763/520

Shweder, R. A., Casagrande, J. B., Fiske, D. W., Greenstone, J. D., Heelas, P., Laboratory of Comparative Human Cognition, and Lancy, D. F. (1977). Likeness and likelihood in everyday thought: magical thinking in judgments about personality [and comments and reply]. *Current anthropology, 18*(4), 637–658. https://doi.org/10.1086/201974

Sinha, C., Sinha, V. D. S., Zinken, J., & Sampaio, W. (2011). When time is not space: The social and linguistic construction of time intervals and temporal event relations in an Amazonian culture. *Language and Cognition, 3*(1), 137–169. DOI: 10.1515/langcog.2011.006

Starmans, R. J. C. M. (2018). The Predicament of Truth: On Statistics, Causality, Physics, and the Philosophy of Science. In M. J. van der Laan, S Rose (Eds.), *Targeted Learning in Data Science* (pp. 561-584). Springer, Cham. https://doi.org/10.1007/978-3-319-65304-4_30

Suppes, P. (2007). Statistical concepts in philosophy of science. *Synthese, 154*, 485–496. https://doi.org/10.1007/s11229-006-9122-0

Veissière, S. P. L., Constant, A., Ramstead, M. J. D., Friston, K. J., & Kirmayer, L. J. (2019). Thinking through other minds: A variational approach to cognition and culture. *The Behavioral and brain sciences, 43*, e90. https://doi.org/10.1017/S0140525X19001213

Wallace, A. F. (1962). Culture and cognition. *Science (New York, N.Y.), 135*(3501), 351–357. https://doi.org/10.1126/science.135.3501.351

Yates, F., & Healy, M. J. (1951). Statistical methods in anthropology. *Nature, 168*(4287), 1116–1117. https://doi.org/10.1038/1681116a0