Math Education: Collaborative Learning for Educational Equity

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Abstract: The purpose of this article is to analyze how collaborative learning can contribute to the creation of inclusive and contextualized environments in the teaching of geometry. Collaborative learning is emerging as a promising approach to promote logical-mathematical reasoning, critical thinking, and equity among diverse student populations. Historical and structural factors significantly impact the physical well-being and mathematical development of many Brazilian students, particularly in the context of plane geometry, which requires mathematical literacy, spatial reasoning, and problem-solving skills that are often hindered by inequitable educational conditions. This study presents an experiential narrative detailing the planning and implementation of a collaborative geometry activity with first-year high school students in Paraíba Valley Metropolitan Area, São Paulo, Brazil. Findings highlight how collaborative approaches promote students’ mathematical cognitive development and address intersectional factors and inequalities in the Brazilian educational landscape. Collaborative activities stimulated student engagement, promoted meaningful interactions, and facilitated inclusivity, allowing each student to construct his or her own knowledge. In addition, this study highlights the importance of ongoing professional development for educators to enhance collaborative practices and create equitable learning environments in mathematics education. These activities were instrumental in promoting active student participation, productive interactions, and inclusive education, thereby enhancing learning opportunities for all students.

Keywords: justice; inclusion; pedagogical practices; empowerment, collaborative learning

1. Introduction

The need for equitable educational approaches has prompted education professionals to seek more dynamic pedagogical methods. Around the world, this movement is paramount to create learning environments that stimulate students’ creativity and enable them to face the complex challenges of the future (Cohen & Lotan, 2014). In the context of mathematics education, collaborative learning (CL) is emerging with promising proposals capable of developing logical-mathematical reasoning and critical thinking among students from different learning cycles.

According to Torres and Irala (2014), CL is based on the premise of the social construction of knowledge, where learning takes place through interaction between individuals, strengthening understanding and developing complex cognitive skills. For Jilk (2016), a deep understanding of this premise contributes to reflecting on the underlying conceptions of the educational process and stimulates more inclusive, equitable and contextualized pedagogical approaches, taking into account the diverse forms of knowledge and the different perspectives of students. For Torres et al. (2004), these activities transcend the mere execution of group tasks and are seen as a resource for equity that articulates the interdependence of participants in problem solving.

Equity is defined as “the quality of being fair and impartial,” highlighting the importance of recognizing and addressing the inequalities that exist in school settings (Lotan, 2022). However, as noted by Van de Walle (2009), factors such as scarcity of educational resources, socio-economic and cultural inequalities, and gender and racial stereotypes can negatively impact students’ equitable access to quality mathematics education. According to Lotan (2022), the adoption of CL strategies mitigates these influences by providing students with the
opportunity to contribute their individual abilities and collaborate in the collective construction of knowledge. This approach, emphasized by Wood et al. (2012) and Stoehr et al. (2017), enhances mathematical literacy and promotes collective reflection, which is fundamental to students’ academic development and to building a more inclusive and collaborative school community.

Ferreira et al. (2023) highlight that historical and structural elements adversely impact the physical well-being and mathematical development of many Brazilian students. This complexity is even greater when considering the importance of plane geometry for students’ cognitive development. According to Ferreira et al. (2023), concepts of angles, lines, and geometric figures provide the basis for understanding space, shape, and measurement, essential skills for solving real-world problems.

Moreover, plane geometry stimulates spatial reasoning, critical thinking, and the ability to interpret visual data, contributing to general mathematical literacy (Boaler, 2019). Engagement with geometric problems also develops logical argumentation and collaborative work skills, essential for forming citizens capable of making informed decisions. In the context of equitable pedagogical practices, the teaching of plane geometry should be accessible to all students, regardless of their socioeconomic or cultural background. Educators must remove barriers to learning, providing equal opportunities to explore and apply geometric concepts. Thus, by integrating inclusive and collaborative pedagogical approaches, as discussed by Cohen and Lotan (2014), teachers can help students develop a solid and practical understanding of geometry, promoting meaningful learning with a positive impact on other areas of knowledge.

But how can CL contribute to the creation of inclusive and contextualized environments in the teaching of geometry that strengthen the mathematical cognitive development of students in different educational settings?

CL enhances inclusive and contextualized geometry instruction and strengthens the mathematical development of students in diverse educational settings. CL promotes diversity and equity by encouraging all students, regardless of background or ability, to actively participate and integrate different perspectives in geometric problem-solving (Silva Richetto et al., 2024). It facilitates knowledge sharing and collective understanding, allowing students to bridge learning gaps and deepen their understanding of geometric concepts while stimulating logical reasoning and critical thinking. Such inclusive environments promote quality mathematics education and prepare students for real-world challenges, thereby promoting greater educational equity. Integrating geometry through CL indirectly supports the Sustainable Development Goals (SDGs), particularly SDG 4 - Quality Education, which aims to provide inclusive, equitable, and quality education and lifelong learning opportunities for all. In addition, by valuing diversity and collaboration, CL supports SDG 5 - Gender Equality by challenging stereotypes and ensuring equal opportunities, and SDG 10 - Reduced Inequalities by reducing socioeconomic and educational disparities.

Although the importance of incorporating geometric concepts in the curriculum is widely recognized for developing essential spatial reasoning and visualization skills, studies conducted by Vereshchagina (2010) indicate that the interdisciplinary or disciplinary integration of geometry has been neglected in some elementary schools. Many students complete their basic education without an understanding of geometry, which may limit their ability to solve complex problems in science and engineering (Cumino et al., 2021). This gap highlights an opportunity for research to explore pedagogical strategies to ensure that all students have access to meaningful and inclusive geometry instruction. Incorporating equitable pedagogical practices can help ensure that all students develop foundational cognitive skills for a more complete adult life, as well as a greater ability to interpret the world around them. In light of this scenario, with the aim of reflecting and internalizing changes in teaching practice, this thesis aims to analyze how CL can contribute to the creation of inclusive and contextualized environments in the teaching of geometry.

To achieve this goal, through an experiential narrative, the planning and implementation of a collaborative activity in a geometry class with first-year high school students in a public state school in Paraíba Valley Metropolitan Area, in the interior of São Paulo State, Brazil, is presented. At the end of this report, after observations and reflections on the development of this pedagogical practice, it is highlighted how cooperative approaches can promote the mathematical cognitive development of students, considering the various intersectional factors and inequalities present in the Brazilian educational scenario.

More directly, the aim of this article is to analyze how Collaborative Learning (CL) can contribute to the creation of inclusive and contextualized environments in the teaching of
geometry. To achieve this goal, this research seeks to answer the following questions: (1) How can CA promote students’ cognitive-mathematical development in different educational contexts? (2) How can the implementation of collaborative activities affect educational equity in geometry education?

2. Materials and Methods

The present master’s project is part of the “Teacher Training for Basic Education” concentration of the Professional Master’s in Education (MPE) at the University of Taubaté (UNITAU), under the research line “Pedagogical Practices for Equity”.

This study falls under the category of experience report (ER), which according to Mussi et al (2021) the integration of equitable activities in mathematics education can improve and strengthen students’ understanding and promote their cognitive development. Thus, the pedagogical narrative is based on pedagogical activities observed and carried out during a mathematics class in basic education. activities observed and carried out during a mathematics class in basic education. The theoretical foundation of the research is based on the work of Van de Walle (2009), Boaler (2022), and Cohen and Lotan (2014), who are recognized for their fundamental contributions to promoting collaborative learning and equity in mathematics education.

The research examines the internalization of changes in classroom practice through the planning and implementation of a CL activity in a mathematics classroom with eighth grade students in a public state school. The pedagogical activity in question was developed during the first semester of the academic year, in a public elementary school located in the central region of Pindamonhangaba, São Paulo State, Brazil.

Pindamonhangaba is located approximately 160 km from São Paulo, the capital of the State of São Paulo, Brazil. With an estimated population of approximately 165,000 inhabitants, the city is known for its rich historical and cultural heritage, preserving distinctive characteristics from Brazil’s colonial period. In recent decades, Pindamonhangaba has experienced significant industrial growth, attracting numerous companies from different sectors, creating jobs and stimulating regional economic development.

The city’s education system includes both public and private schools, providing education from early childhood through high school. However, like many rural areas and small towns in Brazil, Pindamonhangaba faces significant challenges, including the need for adequate educational infrastructure and ongoing teacher training. Understanding these challenges and opportunities is critical to developing educational strategies that promote equity and quality of education in the region. In this context, student participation in this research will ensure a diverse representation of educational contexts, allowing for an inclusive approach to capture a variety of experiences in the pursuit of equity in mathematics education.

The school has 8 classrooms, each with a capacity of 35 students, with two classes for each grade of the last years of primary education. The dynamics of the activity was divided into two classes of 45 minutes each. Due to the high number of participants, 32 students, and the lack of suitable spaces in the school for the planned activity, it was carried out in the classrooms themselves.

Managing a crowded classroom is complex, since classrooms are characterized by multidimensionality, with many events happening at the same time, resulting in a rush to resolve activities. Weinstein and Novod-worsky (2015) report that in order to reduce cases of indiscipline generated by simultaneity, it is necessary to understand that successful classroom management encourages self-discipline and social responsibility, as well as stimulating the teacher-student relationship in favor of effective learning.

The focus of the lessons was to work on the concept of the perpendicular bisector, which is a line that divides a line segment into two equal parts by passing through its midpoint and forming right angles (90 degrees) with it. The perpendicular bisector plays a fundamental role in geometry because it establishes an equidistance between the endpoints of the segment and any point on the line, allowing the solution of various geometric problems. In addition, the concepts of the perpendicular bisector are essential for the accurate construction of regular polygons, circles, and other geometric figures, as well as for understanding symmetries and relationships between different geometric shapes. This approach is designed to strengthen spatial reasoning skills and provide a solid foundation for future mathematical exploration and related fields.

First, a detailed explanation of the concept of perpendicular bisectors was given, following the curriculum, to ensure that students had an adequate foundation for subsequent
work. Then, leveling lessons were conducted to reinforce basic geometry concepts such as finding the midpoint of a line segment and locating points on the Cartesian plane, as more than half of the class showed deficiencies in these concepts.

The approach included individual and collaborative activities that challenged students to locate points on the Cartesian plane and create line segments between specific points. For greater precision and clarity, a projector was used to display the Cartesian plane during the explanation of the activities. GeoGebra software was used to project the Cartesian plane to ensure greater accuracy in representing the concepts covered in the activity. The projection of the Cartesian plane allowed the students to visualize the explanations more accurately and to actively participate in the exercise, working together to identify points and create line segments using the software. This technological approach contributed significantly to the understanding of geometric concepts and provided a more engaging and accurate learning experience.

Students also had the opportunity to actively work on the whiteboard and apply the concepts they were learning. Student mistakes were turned into opportunities for collective learning, encouraging peer correction and valuing peer collaboration. Throughout the process, the teacher documented her observations in a journal to analyze the impact of changes in her teaching practice. Later, these notes were used to evaluate how collaborative learning can promote an equitable and contextualized environment in everyday situations and strengthen students’ mathematical understanding.

To validate the scientific relevance of this experiential report, we followed the steps recommended by Mussi et al. (2021) to establish theoretical and structural assumptions for the development of collaborative reports aimed at knowledge construction. The study employs discourse analysis, following the methodology proposed by Medeiros and Amorim (2017), to interpret the discourse of teacher-researchers in their field journals, exploring content, structure and patterns. This analysis aims to assess the complexity of CL planning, as proposed by Marcondes and Brisola (2014). Triangulation is employed as a methodological measure to ensure the validity and reliability of the findings.

In addition, a reflection was made on how pedagogical practices sensitive to interculturality in geometry education promote equity and inclusion in the educational process. The narrative of experience is presented in the first person, reflecting the activities and experiences lived by the first author of this work. The faces of the students are covered with a gray circle to preserve their identity in the images.

3. Results

In the search for equitable strategies to enhance students’ mathematical understanding, we adopted collaborative learning as a recurring practice in my mathematics classroom, guided by Cohen and Lotan (2014, p. 3), who state that “at some level, students need each other to complete the activity; they cannot do it alone.” Recognizing the importance of balancing individual engagement with group work, we carefully designed an activity that allowed students to collaborate while receiving individual guidance when needed.

Before implementing the activity, we recognized the importance of a differentiated approach to meet the diverse learning needs of students. Inspired by the need to make equity a reality throughout the educational process, we conducted an explanation of the concept of the perpendicular bisector to provide a foundation for further understanding. During this explanation, we encountered a gap in students’ knowledge: difficulty finding the center of a line segment and locating points on the Cartesian plane. So, we decided to review the basics and explain in detail how to locate points and calculate the midpoint.

We then devoted a week of the leveling class to reinforcing these concepts, a total of 4 lessons of 45 minutes each. We instructed students to individually locate points on the Cartesian plane, followed by the task of connecting segments between specific points. Surprisingly, some students struggled to understand what a line segment was. We patiently provided the necessary explanations and encouraged them to persevere. The next day, we used a projector to mirror the Cartesian plane for greater accuracy. Figure 1 shows the activity using GeoGebra software (1a) and a student solving the activity individually on the board (1b).
Figure 1. Activity carried out using GeoGebra software (a) and the students working individually on the board (b).

We worked together to identify points and create segments, with students actively participating by completing tasks on the board while the rest of the class watched attentively and practiced individually in their notebooks. During this activity, some students made mistakes that were used as opportunities for collective learning, encouraging peer correction and valuing collaboration. These mistakes contributed to meaningful student learning because they are part of the learning process and, when viewed as a natural part of it, tend to contribute to the development of understanding. Students can feel comfortable making mistakes, understanding that this is how the teacher identifies points for attention and provides opportunities for growth and development (Boaler, 2019).

This experience not only highlighted the importance of differentiated approaches, but also the positive impact of collaborative learning on the development of students’ mathematical thinking. This approach allowed all students to grasp the concepts of midpoints and segments and aimed for high levels of learning, building on Boaler (2019) premise that everyone can understand and succeed in mathematics with the right message and instruction.

Figure 2 shows the activity carried out collaboratively among the students, demonstrating their autonomy in helping each other during challenging moments.

Through individualized instruction, students were able to overcome their initial difficulties and progress in their mathematical knowledge. In addition, the activity reinforced the idea that school should be a safe environment to explore and make mistakes, fostering a culture of inclusive and empowering learning where students grasp concepts not through rote memorization but by seeking relational understanding, as described by Van de Walle (2009), which involves understanding and applying relationships between different mathematical concepts or situations.

At the end of the activity, students were asked to write a short note describing their
feelings and what they had learned during the group activity. They understood the mathematical concepts covered and emphasized that the way the activity was conducted made them feel valued and supported in their socio-emotional skills. They mentioned that working in groups provided them with a unique opportunity to collaborate, share ideas, and solve problems together, which not only strengthened their understanding of the concepts, but also fostered a positive and enriching learning environment. This feedback underscores the importance of pedagogical practices that not only teach academic content, but also cultivate interpersonal skills that are essential to the holistic development of students.

This feedback from the students deeply touched me, leading to deep reflection on my pedagogical practice. Realizing how the activity helped students understand mathematical concepts, in addition to significantly contributing to the development of their socio-emotional skills, made me consider even more the importance of a learning environment that values not only academic knowledge but also emotional well-being and collaboration among students. This reinforced my conviction that promoting equitable and inclusive education is not just a goal but a fundamental responsibility as an educator.

Expanding critical reflection on the educator’s own role in facilitating collaborative learning involves a deep analysis of the challenges faced and how these challenges can be more effectively addressed in the future. We recognize that managing a classroom with a large number of students and limited resources presents significant challenges. To improve, it is crucial to explore alternatives to optimize the use of available resources and adapt pedagogical strategies that promote equitable participation of all students. This includes, for example, considering different ways of grouping students, providing additional support for students with specific needs, and developing activities that further encourage cooperation and communication among students. Furthermore, reflecting on the effectiveness of the approaches used and continually seeking pedagogical updates and training are essential steps to foster an inclusive and collaborative learning environment.

Reflecting on the equity approaches of Rachel Lotan, Jo Boaler, and Van de Walle, I see how collaborative learning strengthens students’ mathematical understanding and prepares them to meet future challenges with competence and confidence. Equity in mathematics education is not only about ensuring that all students have access to the same learning opportunities, but also about recognizing and valuing diverse forms of knowledge and students’ different perspectives. Equitable pedagogical practices have the power to motivate all students, regardless of background or ability, to reach their full academic potential. Therefore, it is critical for educators to commit to implementing inclusive and contextualized approaches that promote equity and the holistic development of each student. We will continue to incorporate these practices into my teaching, adapting as necessary to meet evolving needs.

4. Discussion

Throughout my journey as an educator, we have always looked for ways to improve my pedagogical approach to better engage students and promote meaningful learning (Boaler, 2022). Recently, we were faced with particular challenges when planning a classroom activity due to a large number of students and inadequate infrastructure outside the school environment. We decided to develop a hands-on activity focusing on mathematical concepts such as Cartesian coordinates, center of gravity, and perpendicular bisectors using GeoGebra software (Van de Walle, 2009). Despite logistical limitations, such as the lack of curtains on the windows, which affected the visibility of the TV screen, we adapted the environment to maximize the use of available resources and to promote collaborative learning.

During the whole activity, we observed a positive change in the classroom dynamics. Students actively participated by sharing ideas and solving problems together (Cohen and Lotan, 2014). For instance, one group of students applied the concept of Cartesian coordinates to map out their school’s floor plan, demonstrating their understanding by correctly identifying key points and plotting them accurately on the software. Another group successfully used the concept of the center of gravity to balance various objects they brought from home, explaining how they determined the balancing point in their presentations. These activities not only evidenced their grasp of the theoretical concepts but also illustrated their ability to relate them to their everyday experiences, which significantly enhanced their engagement and interest in the subject.

This experience reinforced my belief that teaching mathematics is about connecting these concepts to students’ realities, thereby promoting more meaningful learning (Boaler,
The collaborative practice allowed students to organize and apply their acquired knowledge effectively, while also creating an inclusive and equitable learning environment. All students had equal opportunities to actively participate using the same available resources, which contributed to a learning environment where everyone felt valued and encouraged to contribute their individual perspectives (Cohen and Lotan, 2014).

In addition to strengthening their mathematical skills, the activity also promoted the development of socio-emotional competencies such as cooperation and communication, crucial for students’ academic and personal success (Van de Walle, 2009). This is particularly relevant considering the cultural and socioeconomic diversity among students, as pedagogical practices sensitive to these realities can significantly promote equity and inclusion in the educational process.

At the end of the activity, the students were invited to write a short note describing how they felt and what they had learned during the group work. Their responses indicated a deep understanding of the concepts and an appreciation for the inclusive and supportive environment (Boaler, 2019). Many students expressed that working together allowed them to feel more confident in their abilities and that they enjoyed learning in a way that connected the material to real-world applications. This feedback was particularly touching for me as an educator, prompting a profound reflection on the effectiveness of collaborative learning and how it nurtures not just intellectual growth but also socio-emotional development (Cohen and Lotan, 2014).

Reflecting critically on my role as an educator in facilitating collaborative learning, we recognize that my responsibility extends beyond merely delivering content. We must create an environment where students feel empowered to engage actively, share ideas, and collaborate effectively. One challenge we faced was managing the diverse skill levels within the classroom, which required adaptive teaching strategies to ensure that each student could contribute meaningfully (Boaler, 2022). In the future, we plan to address this by incorporating more differentiated instruction techniques and providing additional support for students who may need it. Moreover, we aim to enhance my facilitation skills to better navigate group dynamics and foster a classroom culture that values each student’s unique contributions.

Considering how the lessons learned from this experience can be applied in future activities or in other educational contexts is essential to highlight the sustainable impact of the pedagogical approach adopted. Reflecting on the positive outcomes of this collaborative activity, we aim to integrate even more practices that foster active student participation, encourage critical reflection, and develop socio-emotional skills. This will not only strengthen the learning of mathematical concepts but contribute to a more inclusive and dynamic educational environment. Furthermore, we intend to share these strategies with other educators, promoting the adoption of pedagogical methods that foster equity and collaboration among students, thereby enhancing the positive impact on their academic and personal development.

5. Conclusions

This study uses a descriptive approach, based on a literature review and an experiential narrative, to detail the planning and implementation of a collaborative learning (CL) activity within an eighth-grade mathematics class in a public state school located in the interior of São Paulo, Brazil. The research explores how CL can foster a more inclusive and contextualized learning environment, promote equity, and improve students’ understanding of mathematical concepts.

The literature review and analysis of student engagement underscore the need for pedagogical adaptations and highlight the critical role of professional development for teachers in implementing diverse and collaborative activities aimed at promoting equity and increasing engagement in the geometry classroom. This is essential in the context of the pursuit of educational equity, as it ensures that all students, regardless of background or circumstance, have equal access to high-quality learning experiences that meet their diverse needs and promote their active participation and achievement in mathematics.

The key findings highlight that group activities not only stimulated active student participation and productive interactions, but also facilitated inclusivity, enabling all students to play an active role in constructing their own mathematical knowledge. Analysis of the students’ work revealed diverse perspectives on topics such as the application of logical mathematical concepts, demonstrating the groups’ understanding and reasoning skills on these topics. This is significant in the study because it shows that collaborative learning
environments not only enhance academic engagement, but also promote critical reasoning skills and collaborative problem solving in mathematics. By encouraging students to explore and articulate their solutions to complex mathematical challenges, the study promotes deeper and more collaborative understanding among students, contributing to their holistic development as capable and collaborative learners. These experiences are critical in preparing students to tackle challenging mathematical problems and contribute positively to their learning and the school community.

For future research, it is recommended to include additional assessment methods, such as quizzes or discussion circles, to reinforce learning outcomes and promote equity in the classroom environment. Comparative studies could explore different CL approaches in different school contexts, examining how collaborative activities affect schools with different geographical or socioeconomic profiles. In addition, research on the use of educational technologies, such as simulations or gamification, would provide insights into their effectiveness in enhancing students’ learning experiences. Longitudinal evaluations of these methods could provide valuable insights into their lasting impact on students’ mathematical skills and socio-emotional development.

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